Modeling Stock Return Data using Asymmetric Volatility Models: A Performance Comparison based on the Akaike Information Criterion and Schwarz Criterion

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Abstract

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model has been widely used in time series forecasting especially with asymmetric volatility data. As the generalization of autoregressive conditional heteroskedasticity model, GARCH is known to be more flexible to lag structures. Some enhancements of GARCH models were introduced in literatures, among them are Exponential GARCH (EGARCH), Threshold GARCH (TGARCH) and Asymmetric Power GARCH (APGARCH) models. This paper aims to compare the performance of the three enhancements of the asymmetric volatility models by means of applying the three models to estimate real daily stock return volatility data. The presence of leverage effects in empirical series is investigated. Based on the value of Akaike information and Schwarz criterions, the result showed that the best forecasting model for our daily stock return data is the APARCH model.

Keywords: *Volatility, GARCH, TGARCH, EGARCH, APARCH, AIC and SC.*

I. Introduction

A time series data forecasting model that is now commonly used in economics and known as the Autoregressive Integrated Moving Average [ARIMA(*p*,*d*,*q*)] model [1]. If no differencing is involved, this model is called an Autoregressive Moving Average [ARMA(*p*,*q*)] with *p* and *q* retaining their original meaning and no *d*. The ARIMA model is a linear and symmetric model which is appropriate only for linear and symmetric data [2]. However, one often finds asymmetric volatility time series data to forecast. To resolve such data, Autoregressive Conditional Heteroskedasticity (ARCH) was initially used to model inflation data in the UK which contained asymmetric volatility [3]. This model has been proved suitable for data having asymmetric volatility and short lag structures. The ARCH model was extended to GARCH which is more flexible to lag structures [4]. Both models have symmetrical volatility response characteristics to shocks, either positive or negative shocks. Financial data in particular stocks have asymmetric volatility, i.e. different volatility movements against an increase or decrease in the price of an asset [5]. Some of the models that can also be used to overcome asymmetric volatility problems such are TGARCH, EGARCH and APARCH models.

The TGARCH model has the advantage of measuring the volatility of stock prices with any difference in the effects of positive shocks and negative shocks [6]. For an asymmetric model, the EGARCH model seems more suitable [7]. Then, the APARCH model is used to correct the weaknesses of the ARCH and GARCH models in capturing the asymmetry phenomenon [8].

II. Asymmetric-Garch Family Models

In this section, we review the GARCH models preceded by Autoregressive Moving Average (ARMA) and Autoregressive Conditional Heteroskedasticity (ARCH) models. Then we present briefly the three asymmetric- GARCH family models.

A. ARMA Model

ARMA models provide a good forecast of volatility. An ARMA(*p*,*q*) model is a combination of AR(*p*) and

MA(q) models and is suitable for univariate time series modeling [9-10]. The ARMA(p,q) model can be expressed as:

$$y_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i y_{t-1} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \quad (1)$$

Here the model orders p, q refer to p autoregressive and q moving average terms. This form of model assumes that the time series is stationary. In the absence of a stationary process, the impact of previous values is non-declining. If a process contains a unit root that is non- stationary, and it cannot be modeled as an ARMA model, it instead has to be modeled as an ARIMA.

B. ARCH Model

The Autoregressive Conditional Heteroskedasticity model, also known as ARCH, is useful when the data researched is a non-linear character. One approach used is to include a free variable capable of predicting the volatility of the error [11]. This varied range of errors occurs because the error range is not only a function of the free variable but also depends on the extent of the error in the past [3]. In the cross-section data, the heteroskedasticity that occurs directly related to free variables, so to overcome it only needs to do the transformation of the regression equation. However, in the ARCH model, heteroskedasticity occurs because time series data has high volatility. If a data during a period has a high fluctuation and the error is also high, followed by a period where the fluctuation is low and the error is also low, the error range of the model will depend on the fluctuation of the previous error. If the error range depends on the fluctuation of the quadratic error from some previous period (lag p), then the ARCH model (p) can be expressed in terms of the following equation,

$$\sigma_t^2 = \theta_0 + \theta_1 e_{t-1}^2 + \theta_2 e_{t-2}^2 + \cdots + \theta_p e_{t-p}^2 \quad (2)$$

To check the existence of the effect of asymmetric effect one can use a sign bias test. Another way is by looking at the correlation between standard residual squares of ARMA model with GARCH residual standard lag model using cross correlation. If there is a stem that exceeds the standard deviation or is marked by an asterisk, meaning that bad news and good news conditions have an asymmetrical effect on volatility.

C. GARCH Model

If the ARMA model is assumed to have an error variant, it is recommended to use the GARCH model [4]. In the GARCH model, the error range depends not only on past error but also on the error of the past period [12]. If the error range is affected by the previous period p error (lag p ARCH element) and the error range q of the previous period (lag q GARCH element), then the GARCH model (p, q) can be expressed as:

$$\sigma_t^2 = \theta_0 + \theta_1 e_{t-1}^2 + \theta_p e_{t-p}^2 + \lambda_1 \sigma_{t-1}^2 + \cdots + \lambda_q \sigma_{t-q}^2 \quad (3)$$

D. Three Extensions of GARCH Models

a. EGARCH Model

The EGARCH model has the following form,

$$Ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{e_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \lambda \left[\frac{|e_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \quad (4)$$

where ω, β, γ and λ are the estimated parameters. $Ln\,\sigma_t^2$ is an exponential GARCH model, ω is a parameter of the ARCH model, β is the magnitude of the effect of positive issues on the current variety, γ is the magnitude of the effect of last period's volatility affecting the current variety and λ is a parameter of GARCH model.

b. TGARCH Model

The threshold *GARCH* (TGARCH) model is a development of the model (EGARCH) and the GJR-GARCH model. Given Y_t is the random variable iid (independent identical distribution) with E (Y_t) = 0 and Var (Y_t) = 1. Then (e_t) is called the threshold GARCH process (p, q) if it satisfies an equation of form,

$$\begin{cases} e_t = \sigma_t Y_t \\ \sigma_t = \theta_0 + \sum_{i=1}^{p} \theta_i^{(1)} e_{t-i}^{(1)} - \theta_i^{(2)} e_{t-i}^{(2)} + \sum_{j=1}^{q} \lambda_j \sigma_{t-j} \end{cases} \quad (5)$$

where $e_t^{(1)} = \max(e_t, 0)$, $e_t^{(2)} = \min(e_t, 0)$ dan $e_t = e_t^{(1)} - e_t^{(2)}$ are the effects of the threshold. The variables θ_0, $\theta_i^{(1)}$, $\theta_i^{(2)}$, and λ_i are native numbers [12]. Based on the equation (2.25), the value of σ_t^2 is

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{p} \theta_i e_{t-i}^2 + \gamma_i e_{t-1}^2 d_{(e_{t-i})>0} + \sum_{j=1}^{q} \lambda_j \lambda_j^2 \quad (6)$$

Conditions in the event of good news ($\varepsilon_t > 0$) and bad news ($\varepsilon_t < 0$) give a different effect on the variety. The influence of good news is shown by θ while the influence of bad news is shown by (θ + γ). If γ ≠ 0, then there is an asymmetric effect. The e_t series has an average of zero and no correlation. Let y_t be the observational set during time t, with t = 1, 2, ...,t being influenced by the exogenous variable x_t' where x_t' is the vector of the weak independent variable of size n_t, d is the parameter vector or coefficient of the exogenous variable. Parameters d, θ_0, θ_i, λ_j, and γ_i are parameters in the estimation, whereas γ_i is also a leverage effect.

c. APARCH Model

The APARCH model which is used to improve the weaknesses of ARCH and GARCH models in capturing the asymmetric power of good news and bad news in volatility [8]. Bad news means that information will have a negative impact on the volatility, such as a drastic increase in fuel prices and a sharp rise in inflation. Good news means that information will have a positive impact on the volatility, such as a sharp increase in sales, decreased loan interest rates and business expansion. The general form of the APARCH model (p, q) is:

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{p} \alpha_i(|e_{t-1}| - \gamma_i e_{t-1})^{\delta} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta} \quad (7)$$

and $\omega > 0$, $\delta > 0$, and $-1 < \gamma_i < 1$ and are estimates, δ estimated using Box Cox transform in standard deviation condition, γ_i's are leverage effects. If the leverage effect is positive, meaning that bad news has a stronger influence compared to good news, and vice versa, e_t is the t-th residual data.

E. *Information Criteria*

There are two criteria that can be considered in determining the best model, they are Akaike Information Criterion (AIC) [13-14]. The formula:

$$AIC_C = 2k - 2\ln(\hat{L}) \quad (8)$$

And *Schwarz Criterion (SC)* with formula:

$$SC = \ln(n)\, k - 2\ln(\hat{L}) \quad (9)$$

where $\hat{L} = p(x|\hat{\theta}, M)$, $\hat{\theta}$ are the parameter values that maximize the likelihood function, x = the observed data, n = the number of data points in x, and k = the number of parameters estimated by the model [15]. Both criterions are used to select a model without a test. A model is said to be interconnected from the second model if and only if the collection of independent variables of the first model is part of the independent variable of the second model. In practice, the determination of a best model can be done by looking at the lowest values of AIC and SC.

III. Materials and Methods

The data used in this paper is the daily stock price return data of Indonesian consumers goods company during the period of February 11, 2012 to November 10, 2017. To forecast the best asymmetric volatility models, first we identify the assumption of stationarity of the data graphically and use the Augmented Dickey-Fuller (ADF) test. If the data meet the assumptions, the next step is to forecast the best ARMA(p,q) models that indicate the best Box-Jenkins models in certain lags using Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots. The next step is estimating the best ARMA(p,q) models parameters using Akaike Information (AIC) and Schwarz Criterions (SC) values. Afterward, we test the ARCH effect using ARCH-LM and test the asymmetry in volatility using sign bias test before estimating EGARCH, TGARCH, and APARCH models. Finally, to determine the best asymmetric volatility model, we evaluate the smallest values of the AIC and SC values of the models.

IV. Results and Discussions

A. *Identification*

The identification of the assumption of stationarity of the data graphically shows that the daily stock price return data is stationary either in the mean or variances. However, to ensure the stationary, we do a unit root test (ADF-test) with a hypothesis. The null hypothesis of a unit root is rejected (P-value = 0.0000) which means the data is stationary. We, therefore, conclude that the time series is stationary at the level and we can proceed to model ARMA(p,q).

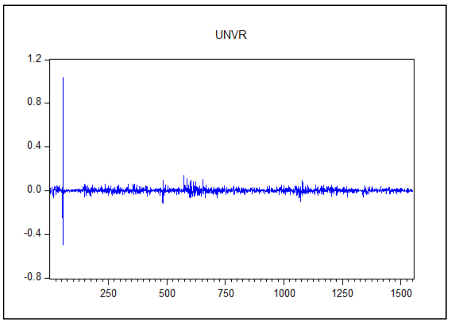
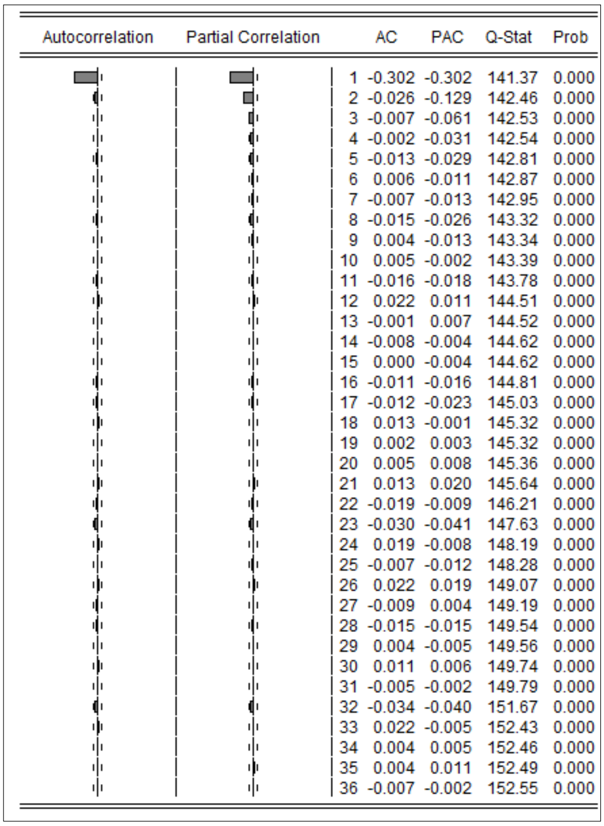


Figure 1. The Plot of the daily stock price return data

B. *Selection of ARMA(p,q)*

To select the best ARMA(p,q), first we plot the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) as shown in Figure ure 2.

The correlogram plot of ACF and PACF shows that lags 1 and 2 are significantly different from other lags. This indicates the best Box-Jenkins models most probably are in those lags. Referring to the plot, evaluation of ARMA (1,0), (0,1), (1,1), ARMA (2,0), ARMA (0,2), and ARMA (2,2) models are carried out.



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.302	-0.302	141.37	0.000
		2	-0.026	-0.129	142.46	0.000
		3	-0.007	-0.061	142.53	0.000
		4	-0.002	-0.031	142.54	0.000
		5	-0.013	-0.029	142.81	0.000
		6	0.006	-0.011	142.87	0.000
		7	-0.007	-0.013	142.95	0.000
		8	-0.015	-0.026	143.32	0.000
		9	0.004	-0.013	143.34	0.000
		10	0.005	-0.002	143.39	0.000
		11	-0.016	-0.018	143.78	0.000
		12	0.022	0.011	144.51	0.000
		13	-0.001	0.007	144.52	0.000
		14	-0.008	-0.004	144.62	0.000
		15	0.000	-0.004	144.62	0.000
		16	-0.011	-0.016	144.81	0.000
		17	-0.012	-0.023	145.03	0.000
		18	0.013	-0.001	145.32	0.000
		19	0.002	0.003	145.32	0.000
		20	0.005	0.008	145.36	0.000
		21	0.013	0.020	145.64	0.000
		22	-0.019	-0.009	146.21	0.000
		23	-0.030	-0.041	147.63	0.000
		24	0.019	-0.008	148.19	0.000
		25	-0.007	-0.012	148.28	0.000
		26	0.022	0.019	149.07	0.000
		27	-0.009	0.004	149.19	0.000
		28	-0.015	-0.015	149.54	0.000
		29	0.004	-0.005	149.56	0.000
		30	0.011	0.006	149.74	0.000
		31	-0.005	-0.002	149.79	0.000
		32	-0.034	-0.040	151.67	0.000
		33	0.022	-0.005	152.43	0.000
		34	0.004	0.005	152.46	0.000
		35	0.004	0.011	152.49	0.000
		36	-0.007	-0.002	152.55	0.000

Figure 2. Correlogram plot.

We use AIC and SC to select the best parameters *p* and *q* of ARMA to fit in the series. The result of the ARMA(*p*,*q*) selection models are shown in Table 1. The table shows that all parameters for ARIMA (0,1) are significant and has the lowest values of AIC and SC compared to other models. This indicates that the best-suited model for the mean equation is an ARMA (0,1) model for all time series. To ensure the result, a residual correlogram was used and show that ARMA(0,1) is really the best model among all the ARMA(*p*,*q*) models.

Table 1. Selection of ARMA(*p*,*q*)

No.	Model	Para meter	Para meter Estimate	P-Value	AIC	SC
1	ARMA (1,0)	β_1	-0.300011	0.0000	-3.92939	-3.92594
		β_1	-0.301622	0.0000		
2	ARMA (0,1)	α_1	-0.360420	0.0000	-3.94919	-3.94574
		α_1	-0.365618	0.0000		
5	ARMA (1,1)	β_1	0.106622	0.1232	-3.94896	-3.94206
		α_1	-0.454168	0.0000		
		α_1	-0.472038	0.0000		
7	ARMA (2,0)	β_1	-0.338130	0.0000	-3.94366	-3.93676
		β_2	-0.126629	0.0000		
		β_1	-0.340764	0.0000		
		β_2	-0.129243	0.0000		
9	ARMA (0,2)	α_1	-0.348563	0.0000	-3.94918	-3.94229
		α_2	-0.036189	0.1543		
		α_1	-0.352629	0.0000		
		α_2	-0.365618	0.1112		
11	ARMA (2,2)	β_1	0.907156	0.0070	-3.94622	-3.93242
		β_2	-0.058230	0.5323		
		α_1	-1.255655	0.002		
		α_2	0.342298	0.0607		

C. ARCH and GARCH Test

In the next step, we test the ARCH effect using The Lagrange Multiplier (LM) test. The result is presented in Table 2.

Tabel 2. ARCH-LM effect

Statistics/Probability	Values
F-statistic	295.8919
Obs*R-squared	248.7307
Prob. F(1,1192)	0.0000
Prob. Chi-Square(1)	0.0000

By looking at the probability of χ^2-statistic of ARCH-LM test (p-value = 0.0000), it can be concluded that the squared residuals from previous lags are correlated with the squared residual at time *t*. This indicates the existence of Heteroskedasticity in the data. As a result, the GARCH model can be used to the data. From the evaluation of GARCH models based on the AIC and SC values, the results show that the GARCH(1,0) model has all parameters significant and lowest values of AIC and SC. This indicates that GARCH(1,0) is better than others. Diagnostic checking for GARCH(1,0) model using Ljung Box-Pierce gives significant results with *p*-value >0,5 which indicating the model is appropriate. Therefore, the GARCH(1,0) model is good for making a better estimate for the data.

In the following step, we evaluate the presence of volatility in the data using a sign bias test. The analysis of the sign bias test has *p*-value=0.0000. The null hypothesis of the test was rejected. It can be concluded that positive and negative shocks impact the volatility differently. Asymmetric GARCH models could, therefore, perform well in explaining conditional volatility for the data. The usage of an asymmetric GARCH model is hence justified by the test.

D. Estimation and Comparison of Volatility Asymmetric Models

Estimation of a series of asymmetric GARCH-family models to explain conditional variance and volatility clustering using Ljung-Box on various lags gives a result of EGARCH(1,1), TGARCH(1,1) and APARCH(1,3) are best three models among all models in the lags. All parameters of EGARCH(1,1) having *p*-value<0.01. A similar result for the estimation of the TGARCH model gives all parameters of TGARCH(1,1) having *p*-value<0.01. Estimation of the APARCH model gives the parameters of APARCH (1,3) having *p*-value<0.01. This indicates that

EGARCH(1,1), TGARCH(1,1), APARCH (1,3) models are appropriate for forecasting the data.

To compare the best performance of TGARCH(1,1), EGARCH(1,1)and APARCH (1,3) models, AIC and SC are used. The summary of the comparative performance of the three models is presented in Table 3. Based on the values of AIC and SC, it can be concluded that APARCH(1,3) model outperforms the other models since it has a statistically significant estimation of all parameters and smallest AIC and SC values.

Table 3. Comparison of performance asymmetric volatility models

No	Model	Parameter	Coefficient	P-value	AIC	SC
1	EGARCH (1,1)	ω	-11.6172	0.0000	-4.67259	-4.65536
		θ_1	0.09259	0.0000		
		γ_1	-0.23949	0.0000		
		λ_1	-0.54011	0.0000		
2	TGARCH (1,1)	ω	5.14E-5	0.0000	--4.78279	-4.76554
		θ_1	0.15533	0.0000		
		γ_1	0.52850	0.0000		
		λ_1	0.67815	0.0000		
3	APARCH (1,3)	ω	0.00067	0.0000	-4.84165	-4.81407
		α_1	0.23477	0.0000		
		γ_1	0.30894	0.0000		
		β_1	1.43225	0.0000		
		β_2	-0.77521	0.0000		
		β_3	0.14983	0.0000		
		δ	1.20962	0.0000		

IV. CONCLUSION

The asymmetric volatility models such as APARCH, EGARCH, and TGARCH are suitable for modeling the volatility data. In this study, the APARCH (1,3) is more suitable than EGARCH(1,1) and TGARCH(1,1) for modeling the daily stock price return data of Indonesian consumers goods company since it has the lowest AIC and SC scores and has all statistically significant estimation parameters.

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