Enhancing Visual Abilities in Solving Mathematics Problems

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Abstract

This research is an experiment of categorizing visual abilities by representations of a solving problem through class experiment. The research is to investigate, explore, and enhance geometry thinking in solving mathematics problems presented visually. The tasks are cognitive strategies that rely on the visual representations and the genesis of the abilities with the assumption that the performance requires the manipulation of the visual representations. The model of the research developed from the students' imagination of a visual problem or situations for accomplishing the solution and called binocular rivalry. The data are of interconnection for promoting visual abilities or senses, as the flexibility of thinking and the likelihood of having representations available in prior knowledge. The data analyzed by categorizing according to the visual abilities and by Control Chart of Attributes. The results are: 80% of the visual abilities are nonconforming index; the variation of visual representations is high in nonconforming index, but the depth struggle by the subjects and 70% select the general geometric figures that recognized. The conclusions are comprehensive, and the visual representations are nonconforming index, and the visual senses enhance in all categories.

Keywords: *Visual representation; Visual ability; Categorize visual skills; Geometry thinking; Cognitive strategy.*

I. INTRODUCTION

Students and lecturers are mainly using algebra representation in solving a mathematics problem that presented visually. The students are more likely to engage with the material grounded in memorizing or recalling. Mathematics is more than just algebraic thinking, and experiences based only on the utilitarian value of mathematics denuded of much of the richness of the discipline. That is algebraic thinking, namely working in symbols manipulated and primarily based on recalling and the strength of memorization. Generally, the solutions come from the recognized knowledge and without a 'new' strategy. While the visual sense or ability based on the thinking in high order, a solution can emerge numerous strategies in solving the problems. So, that is crucial at this time to consider the impact of the visual-based approach to mathematics education.

But, during two years in teaching mathematics at the Education Department, the researcher discovers: (1) the students draw a visual even though no need; (2) there are no visual activities from problem represented visually; (3) avoid the visual representation; and (4) no or little visual activities. In such cases, the visual representations are of unconscious mental imagery. The researcher observed that geometric shapes of the test filtered by the theoretical of the measure instruments because of a mathematics term. For example, the students ask to find a directed line segment equation that presented at the grid Cartesian Coordinates System, and all of them only used a formula without considered the visual behave. The students want to use a formula to find the equation, but the available information that presented visually actually cannot bring them to an answer at all. In-class discussion, the researcher understands that in visual representation, we need a sensitivity or *visual sense*.

The visual sense is one of the abilities of thinking. That is mental imagery when the students manipulate a visual in many representations, focused on the performance to achieve a transition from geometry to the next visual representation. So, the students learn more than just mathematical concepts and skills and involved at more than just a cognitive level [1]. Furthermore, [2] stated that "how a person learns a particular set of knowledge and skills and the situation in which a person learns, become a fundamental part of what learned." In this case, it's essential to explore mathematical learning experimentally, because it includes the material and the dimensions bring to the classroom.

Problems of visual representation have tenaciously persisted that concerns about mathematical literacy, attitudes, and dispositions. It hoped an investigation as the learning culture development of new insights into students' success in mathematics. There is a doubt between a geometric and algebraic concept in the test, i.e., stressing more on algebra or symbolic than the visual perceived. Despite the doubts of the representations considering mathematical activities, it rarely discussed in classes while algebra courses. According to [3], that is a perception of mental imagery because the case is a visual representation.

II. Materials and Methods

The *visual abilities* are learning conditions by doing math, without the influence of determined of actual conditions. That is, the use of problems leads to the construction of doing the math to render the carrying out the process of the sense. The situations, the representations of mathematics can be thought of as systems of interaction to be able to act. For example, the study of geometry objects that based on visualization assets of visual thinking.

A. Materials

The visual representation stayed on mental imagery [4], spatial properties, and considered picture ([5], [1]). The representation is a quasi-pictorial, but an account of mental imagery [6], a computational model, and a neurological one [7], won the analog-propositional [4] because its notion used ambiguously and incoherently.

The visual representation is a mental investigation based on thinking. That is to manage mathematical knowledge into a series of representation thinking. The development stressed by counter-intuitive thinking.

The visual representation is qualitative thinking of geometry objects [2]. It pertains to thinking activities, relatively stable proceeds within local instability, and part of the epistemological and axiological manner. That is, a belief tends to identify, to determine, and to the movements of thought.

The epistemological content refers to the genealogy of concepts. The development based on observation of the problem-solving through series of visual representation. That is fragment of the visual from a conceptual viewpoint [8] when organizing it.

The visual representation is tools to transfer from algebra to another and organized experiences in mathematical activity. The representation system is the process corresponding to a perception or intuition as a cognitive term - the evidence to be seen analogically by representing the geometry objects that look simpler than the algebraic expression. The system is to estimate the solution. For example, an intuition of intersection lines tends towards underestimation of the particular condition. In the idea, the ability to discriminate between many conditions or cases changes with their perceptions. The observations led to the cognition assuming the existence of representation in the shape of a mental image.

The role of the visual representations confirmed on the effect and to indicate the flexibility of the thinking. The route intervenes the intuitions, a high level of doing the mathematics before cognitive praxes, and the geometrical judgment of well-ordering. The judgment of the geometric objects organized as an insight or gestalt idea and the visual representation as *well-ordering thinking*.

Visual thinking is a proto-mathematical that related to the problems. That is a geometric trajectory, the constitution of its memory, the latter being abstract as it is the memory of a prediction. The memory of a prediction is abstract in the double sense of being independent of the context, the invariant that matters for future pretension, away from the specific context of the original action. The structure is the creation of a boundary, which is drawing and complete of non-existing visual on the problems.

B. Method

The research is a study of visual abilities in a method of multidimensional scaling. The purpose is to obtain the representation that showing interactions between the visual and the thinking. This is a neuroimaging studies dedicated to spatial cognition of numerical magnitude [9] (Hubbard et al., 2005).

The investigation contributed to epistemological questions, where the visual is an argument against the thinking as a logical machine. Through teaching and learning, we investigate, explore, and enhance the visual ability when solving mathematics problems. The observation is to investigate the visual representation in solving mathematics problems, guides by the visual ideas. The test items are mathematics problems presented visually abilities, and the answers categorized into the thinking.

Model of the thinking developed from the visual imagination of the problems for accomplishing the solution. That is to a mental imagination in the way to do it in continuity before lost the visual abilities.

Data collected during class activities and analyzed qualitatively after the categorizing according to the visual abilities or thinking. The explanations of the invented of the categorizing data using logic, and the conclusions make the truth statements. Statistically, the invented data analyzed by *Control Chart of Attributes*.

The final elaboration refers first to the conceptually simultaneous role of the visual representation. The isomorphism proposed is between the experience of the inertial movement of the algebraic and the imagery, which precedes subjective movement. The direction and the conceptual come to propose a continuous thinking over the reality in the visual.

III. Results and Discussions

After construct validity with Cramer C = 0.82, we get two items as depicted at Fig. 1 and Fig. 2.

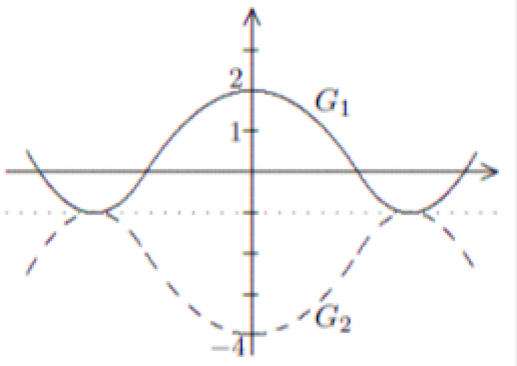

Figure 1. A visual representation to relate G_1 and G_2

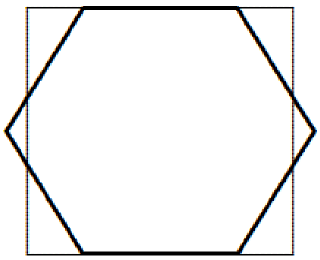

Figure 2. Representation of two intersection pictures

A. Results

Problem 1 of Fig. 1 is modification for improving the visual thinking, to reveal the strategy, and to reduce algebraic expression in constructing visual solution.

The logical connection is a composition of transformations. The strategy is to visualize - G_1 by reflection and then turn down 2 units. The algebraic thinking by matrix transformation is still difficult.

The solutions that G_2 = -G_1 and G_2 = - G_1 – 1 showed that the visual thinking is not complete. Their algebraic thinking seems in organizing algebraic expression. The students also promote a practical way, in using formula or procedure. That is an operation in matrix for getting a solution.

The visual thinking changed gradually from the algebraic. Invented that the trend in series are: (1) $G_2 = G_1^{/}$ or the derivative; (2) G_2 = -G_1 using simple graph $y = x + 1$; (3) G_2 = -G_1 – 1 with two expressions; (4) $G_2 = G_1 - 1$; and (5) G_2 is equal to the invers of – (G + 1).

The solutions deal with simple geometrical representation reduces the algebraic. That is simple for constructing a solution. For an example, transforming the representation to a recognizable one and bring to the solution.

Algebraic expression arranged by a matrix, or transformation of two order matrix only taking a point before and after the transformation. That is a linear transformation, and come to incorrect answer. It is a symbolic expression that precedes and leads to the intervention of the solution.

About 80% respondents solve the problems by formulating algebraic expressions taking from the visual. They connect it and get a new algebraic equation. So, there is a model and then solve the problems algebraically. About 70% of them recognize the visual clearly and never presented again in solving the problems.

About 10% try to solve the problems by illustrating the visual using some available points. They formulate the equation of G_1 and then relate it to the G_2. The solution comes from the modifying one. But, that is hard work, so no good solution at all.

After the teaching and learning, about 10% of the respondents already show the visual abilities. They transform the G_1 by reflecting it to the line y = 0. So, they get another visual but simply than the G_1. The next manipulating is a simply translation.

The problem of Fig. 2 is to determine the area of the intersection of both figures. The problem is of the figures. That is, solving without using the outside visuals (two small triangles), relating the visuals using algebraic thinking and only of 1 minus the two outside triangles, and the unrelated of formula of the area of hexagon to the entire equation arranged for each of visual cases.

When starting with a square of side 1, then a regular hexagon is constructed, concentric with the square, then they use all of the representations holistically.

When the geometric shapes manipulated or need recognizable parts, by experimentation, come to a certain extent [10]. In this research, that is visual representation carried into a solution. The visual thinking is the reflections of abstract concepts and symbols in the problems. The argumentation is also that the visual situations can be used for an insight where the problems are expectations to produce an answer independently from the representations.

This is a problem of shapes and drawings. The visualization is built between the enhancing of thinking and the thinking of representations. Reference

([1], [11]) ensure that a perception of a drawing operations and concepts in mathematics lead to mental imagery.

The students can take information of the problems and retain it by arranging and schematizing the solutions in the representations manner. That supports the visualization that will enhance comprehension and creativity in mathematics education. The visual thinking helps to improve students' thinking in solving the problems. Reference [12] report that visual model in problem solving facilitates students' comprehension and creates solution-finding opportunities.

Reference [13] argues that visualization is much more effective than conventional approaches in strengthening students' intuitions and facilitating learning. Reference [14] considers visualization as a tool which serves to attract students' attention by drawing geometric concepts and models with varying effects to implicate the presence of various mathematical systems and various spaces; to help individuals acquire the habit of abstraction and thus improve their cognitive independence and productivity; to ensure meaningful learning and retention of information. In this research, underline the fact that visualization is not only a tool but model of thinking beginning at problem solving activities or presenting a proof.

The pattern of the algebraic of Problem 2 comes from an adapting strategy. The strategy is to have an action in every representation. The steps of the thinking are drawing, making relationships, and testing a solution. The steps are for determining from the available information in a problem. In mind mapping, a series of visual representation could be represented algebraically or visually. That is of a relation between operations and the implicit one, so the logical condition of a problem can be seen similar to a certain problem.

B. Discussions

The students accept the visual as an immediately and intuitively sound and perfectly *robust* statement. The visual abilities correspond to the thinking, i.e. completing, constructing, and manipulating. But, occurring in different parts of the brain and involve different memory networks, so do not correspond to the visual thinking.

The mathematics concept developed by simple illustration and the representation of the symbol at almost all. Of the Problem 1, the evidence comes from an ability to imagine visually, rather than having unconscious mental images, and failing to attend to the solution. In this case, most of respondents take a *sinus function* instead of G_1, but become more complicated. While the visual ability come more easy with a reflection of line $y = 0$ and then a simple translation.

Of the Problem 2, the visual abilities emerge in the reduction of the representation, namely present more algebraic competency than the visual, and they do it in symbolic equations. When visual activity arose firstly, they solve the problem without continuing the visual.

When threated to the consistence of the visual representation and the thinking, the intersection of the algebraic concept changed to the visual representation. According to the logical approach, the flexibility of the cognitive illustrates the visual for imaging the recognized one. For the type of student materials that is 20% need a visual solution, and 80% need an evaluation of the practical skills doing.

The relation between algebra representation, the visual and the abilities in solving the two problems presented explained in Table I.

Table I. A Mapping of Algebraic and Visual Abilities

Mapping Competencies		
Algebraic Representation	Visual Representation	Visual Abilities
Formulating	Formulating	Transforming
Connecting	Relating	Simplifying
Modelling	Modifying	Manipulating

The students' visual thinking is responses to transform the representations and alternately using algebra. Firstly, they used more algebra knowledge, i.e., an algorithmic point of view. The next level is to simplify critical attributes of the geometric shapes. In this case, the students connect the visual situation to the algebraic concepts and relating the situation by the algebraic to solve the problems.

In mathematics, the critical attributes stem from the concept definition [15]. For example, for the intersection of geometric figures, there are three: used the only one figure in the intersection of two figures, using information that given, and simplifying the visual representation.

Mathematics reflections of geometric concepts and shapes in the problems presented visually are particular abilities. The visualization can be thought as a phenomenon that could introduce experimentally to a certain extent [16]. In representing algebraically or visually into a solution, the thinking could be seen as the reflections of abstract concepts and symbols in the problems.

The argumentation is also that the visual situations can be used for an insight where the problems are expectations of the students to produce an answer independently from the representations. This is that the visual through shapes and drawings the aspect of mathematics. The visualization is built between the enhancing of thinking and the thinking of representations.

The students can take information of the problems and retain it by arranging and schematizing the solutions in the representations manner. So, the visualization enhances comprehension and creativity, and the visual thinking helps to solve the problems. Reference [11] report that visual model in problem solving facilitates students' comprehension and creates solution-finding opportunities.

Reference [14] argues that visualization is much more effective than conventional approaches in strengthening students' intuitions and facilitating learning. Reference [13] considers visualization as a tool which serves to attract students' attention by drawing geometric concepts and models with varying effects to implicate the presence of various mathematical systems and various spaces; to help individuals acquire the habit of abstraction and thus improve their cognitive independence and productivity; to ensure meaningful learning and retention of information. In this research, underline the fact that visualization is not only a tool but model of thinking.

IV. Conclusions

The growing of visual thinking can be raised through the representation. This is regarding the students and serve to foster much needed of the manipulation. The analysis shows that the visual thinking varies to grasp the visual sense. The students want to solve algebraically, but the visual sense required because of their learning habits.

The development looks like the presented exercises in many textbooks and lecture, but ought to change by thinking experience when working in geometry. For teaching materials, the mathematics class alike to declare experiencing obstacles in the algebraic approach contained in the text or courses. The material should use more for geometry terms.

The visualizing abilities need 'to draw', 'to use', 'to manipulate', 'to construct', and 'to explore' in solving the problems. We need form of etymological material for extending the visual representation.

For teaching materials and evaluation based on visual thinking analysis, concluded that the students geometric skills. The exercise is used to improve students' visual competency. In terms of technical solving the mathematics, there is no difference of the difficulty but should be done and studied in class.

For the evaluation material, this type of evaluation will be more emphasized by visualizing as it is adapted to the learning. For the test techniques requiring individual and group evaluation, and require varying evaluations between formal logic and the etymologic.

Prepare the conclusion in a single paragraph without reference. Conclusion should be prepared in a complete sentence and paragraph without numbers or pointers. Please inform the future study or application of the current findings.

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